The Hydraulic Geometry of Stream Channels and Some Physiographic Implications

By LUNA B. LEOPOLD *and* THOMAS MADDOCK, JR.

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Quantitative measurement of some of the hydraulic factors that help to determine the shape of natural stream channels: depth, width, velocity, and suspended load, and how they vary with discharge as simple power functions. Their interrelations are described by the term "hydraulic geometry."

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ABSTRACT

Some hydraulic characteristics of stream channels—depth, width, velocity, and suspended load—are measured quantitatively and vary with discharge as simple power functions at a given river cross section. Similar variations in relation to discharge exist among the cross sections along the length of a river under the condition that discharge at all points is equal in frequency of occurrence. The functions derived for a given cross section and among various cross sections along the river differ only in numerical values of coefficients and exponents. These functions are:

$$w=aQ^b$$

$$d=cQ^f$$

$$v=kQ^m$$

$$L=pQ^j$$

where

w = width

d = mean depth

v = mean velocity

L = suspended-sediment load, in units of weight per unit time

Q = water discharge, in cubic feet per second (cfs); a, c, k, p, b, f, m, and j are numerical constants.

These relationships at a given channel cross section and downstream when plotted on graphs are greatly similar even for river systems very different in physiographic setting. The relationships are described by the term "hydraulic geometry."

In the data studied it appeared that when discharges are of equal frequency at different points along a river, that is, equalled or exceeded the same percent of time, the velocity as well as the width and depth of flow, increases with discharge downstream. This increase of velocity downstream results from the fact that the increase in depth overcompensates for the decrease in slope. The tendency for velocity to increase downstream exists on most streams despite the decreasing particle size downstream. This indicates that the mean velocity in a given reach is not merely a function of the size of particles which must be transported but is governed by a more complex interaction among several factors.

Measurements of suspended-sediment load are available for a number of different river cross sections. The suspended load is shown to be an index to total load for purposes of explaining the observed average characteristics of natural channel systems.

An empiric quantitative relation among average measurements of width, depth, velocity, discharge, and suspended-sediment load is derived from data on natural rivers. This relation shows that depth and width, as well as velocity, are functions of the load transported in the channel.

In a simplified example, if the discharge is constant, the empiric relation is: At a given discharge, an increase in width at constant velocity is accompanied by a decrease in suspended-sediment load; conversely, at constant discharge and velocity, an increase in width is accompanied by an increase in bed load.

The writers, like many others, postulate that both discharge and sediment load are factors essentially independent of the stream channel and depend on the nature of the drainage basin. The typical relations between suspended-sediment load and water discharge are shown in quantitative terms. Specifically, at a given stream cross section, suspended-sediment load per unit of discharge characteristically increases rapidly with increased discharge. However, suspended load per unit volume of water tends to decrease slightly downstream.

These characteristics are important determinants of the shape of the cross section of a channel and the progressive changes in its shape downstream. The empiric relation between hydraulic characteristics of the channel and suspended load provides, in semiquantitative terms, a logical explanation of the observed channel shape.

The hydraulic geometry of river channels is presented for several river systems. It is shown that similar equations apply both to rivers and to stable ("regime") irrigation canals which neither scour nor aggrade their beds. The analogy demonstrates that the average river channel-system tends to develop in a way to produce an approximate equilibrium between the channel and the water and sediment it must transport. This approximate equilibrium appears to exist even in headward ungraded tributaries and in a given cross section for all discharges up to the bankful stage.

GENERAL STATEMENT

Geomorphology, that branch of geology dealing with land forms, their genesis and history, has been classically treated almost exclusively in a qualitative manner. William Morris Davis, for pedagogic reasons, invented words and contrived similes which were descriptive and easy to remember, but were supported by intricately developed general argument rather than by field data. The qualitative approach to geomorphology has indeed been constructive, but it would be desirable to analyze some of the concepts quantitatively.

There is available a mass of data on streamflow collected over a period of seventy years representing rivers all over the United States. The field procedure used by engineers to measure the rate of water discharge in a river furnishes concurrent measurements of mean velocity,width, shape and area of the cross section, as well as discharge. Moreover, in the last decade width, depth, and velocity increase downstream with discharge in the form of simple power functions,

$$w=aQ^b \tag{1}$$

$$d=cQ^f \tag{2}$$

$$v=kQ^m \tag{3}$$

The coefficients and exponents are assigned the same letters as for a given cross section of a river, but the values will be different for points in a downstream direction from those for the given section. Average values of the exponents for river basins studied are

$$b=0.5$$

$$f=0.4$$

$$m=0.1$$

To recapitulate, both at a given river cross section and in the downstream direction at mean annual discharge, width, depth, and velocity increase with discharge. The functions representing these relations are similar but involve different exponents and numerical constants.

RELATION OF CHANNEL SHAPE TO FREQUENCY OF DISCHARGE

Thus far the changes in hydraulic characteristics downstream in a river system have been considered only for a discharge equal to the mean annual rate at each point. Comparison of the data already presented will now be made with similar data for infrequent flows.

Computation of the frequencies of various discharges at a gaging station has been discussed. The computation is a somewhat laborious task. Complete cumulative frequency (flow-duration) curves for all stream-gaging stations in a watershed are available for only a few localities; one of the best collections of such data is that by Cross and Bernhagen (1949) for Ohio.

It was desired to avoid choosing streams regulated by many reservoirs, and the Maumee and Scioto River basins were selected as being freer of regulation than most major river systems in Ohio. Duration (cumulative frequency) curves were available for 19 gaging stations in these two watersheds. Though the Maumee River flows northeast into Lake Erie and the Scioto south into the Ohio River, the two basins have a common divide, and study of the data showed no significant differences in the hydraulic characteristics for the purposes of this paper.

At each gaging station discharges that were equalled or exceeded 1, 4, 10, 30, and 50 percent of the time, as well as the mean annual discharge, were selected for study. For each of these discharges, the corresponding velocity, depth, and width were determined by analysis of current-meter data, as explained for figure 3. For each discharge frequency, curves were plotted representing the change of hydraulic factors with discharge in the downstream direction. Figure 10 presents the individual plots of six curves in order to demonstrate the relative goodness of fit. It will be seen that the gaging stations available for the Maumee and Scioto basins did not represent so large a range in discharges as did some of the rivers discussed earlier, and that the points did not fall in so good alinement as in some other basins studied. The width-to-discharge relations are the best; more scatter of points appears on the depth and velocity curves.

For the Maumee and Scioto basins combined the curves of discharge in relation to width, depth, and velocity for each of five frequencies are shown in figure 11.

There is a general parallelism within each family of curves, but this is somewhat more consistent in the width-to-discharge than in the velocity-to-discharge relation. In these data it appears that, with less frequent discharges (flood flows), the velocity increases somewhat faster with discharge downstream than it does in low flows. A similar study made for flood flows in some tributaries to the Yellowstone River in Wyoming showed that velocity remained nearly constant downstream despite a rapid decrease in slope.

It can be seen that there is a pattern of interrelationship of stream characteristics which includes relative frequency of discharge, and it is now possible to define important aspects of the hydraulic geometry of stream channels. An understanding of this geometry is basic to consideration of the adjustment of channel shape to carry the sediment load supplied to the streams. The relation of at-a-station characteristics to the downstream characteristics will now be discussed.

In basin A of figure 12 the line A—C shows the increase of width with increase of discharge at an individual gaging station near the headwaters of a stream, and B—D shows the same relationship for a downstream station. If the discharge A is of the same cumulative frequency (say 50 percent of the time) at the headwater gage as discharge B at the downstream gage, then the line A—B is the increase of width with discharge downstream, corresponding to a cumulative frequency of 50 percent. Also if C and D have the same cumulative frequency (1 percent of the time), then line C—D is the increase of width downstream at a discharge frequency of 1 percent. Similarly then, E—G is the increase of depth with discharge at the upstream gage, and E—F is the increase of depth downstream for a frequency of 50 percent. In the example under discussion, the curves are drawn so that at any frequency the width-to-depth ratio remains constant downstream; that is, A—B is parallel to E—F.

It was noted earlier that because

$$Q=wdv$$

and $w=aQ^b$, $d=cQ^f$, and $v=kQ^m$, then $b+f+m=1.0$